# Dimensionality influence on energy, enstrophy and passive scalar transport.

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The advection of a passive substance by a turbulent flow is important in many natural and engineering contexts, e.g. turbulent mixing, combustion, pollution dispersal, in which the prediction of mixing and dispersion rates of a scalar contaminant is a problem of great interest. Although the concentration of a passive substance exhibits a complex behaviour that shows many phenomenological parallels with the behaviour of the turbulent velocity field, the statistical properties of 'passive scalar' turbulence, strongly influenced by the Kolmogorov cascade phenomenology, are in part decoupled from those of the underlying velocity field [1]. The subject, in the last years, is in fact undergoing a reinterpretation as empirical evidence shows that local isotropy, both at the inertial and dissipation scales, is violated [2]. Moreover, dispersion usually occurs in time dependent inhomogeneous flows, which present a much more complicate behaviour than homogeneous flows and thus are beyond the reach of analytical models or even numerical simulations. In this work, we go one step beyond the homogeneous and isotropic turbulence and consider the simplest inhomogeneous and shearfree turbulent Navier-Stokes motion: the system where an energetic turbulent isotropic field is left to convectively diffuse into a low energy one. In this system the region where the two turbulences interact is preceded by a highly intermittent thin layer - where the energy flux is maximum - that propagates into the low energy region, see [3, 4, 5].

## Transport and intermittency un shearless mixings side

The initial conditions introduce a kinetic energy gradient in the direction of inhomogeneity. Outside this inhomogeneous region, the kinetic energy in the two turbulence fields decays as a power law, with an exponent approximately equal to -1.2, in the three-dimensional simulations, while in two dimensions there is no significant decay. In both cases, the initially imposed energy ratio is preserved during the time evolution of the flow.

The mixing layer between the two homogeneous flows was observed to be highly intermittent, as shown by the large values of the velocity skewness and kurtosis, see [6, 3, 4, 5]. In this work, we focus on the scalar dispersion through the shearless mixing layer.

The scalar interface is spread by the turbulent eddies and a scalar mixing region with high scalar variance is generated in between the two homogeneous flows. The width of this region can be measured by considering the mean scalar distributions. The scalar mixing layer thickness $\Delta_\theta$ is defined, in analogy with the energy layer thickness $\Delta_E$, as the distance between the points with means scalar $\overline{\theta}$ equal to 0.75 and 0.25. After an initial transient of about one eddy-turnover time, the time evolution of these interaction widths follows the one observed for the self-diffusion

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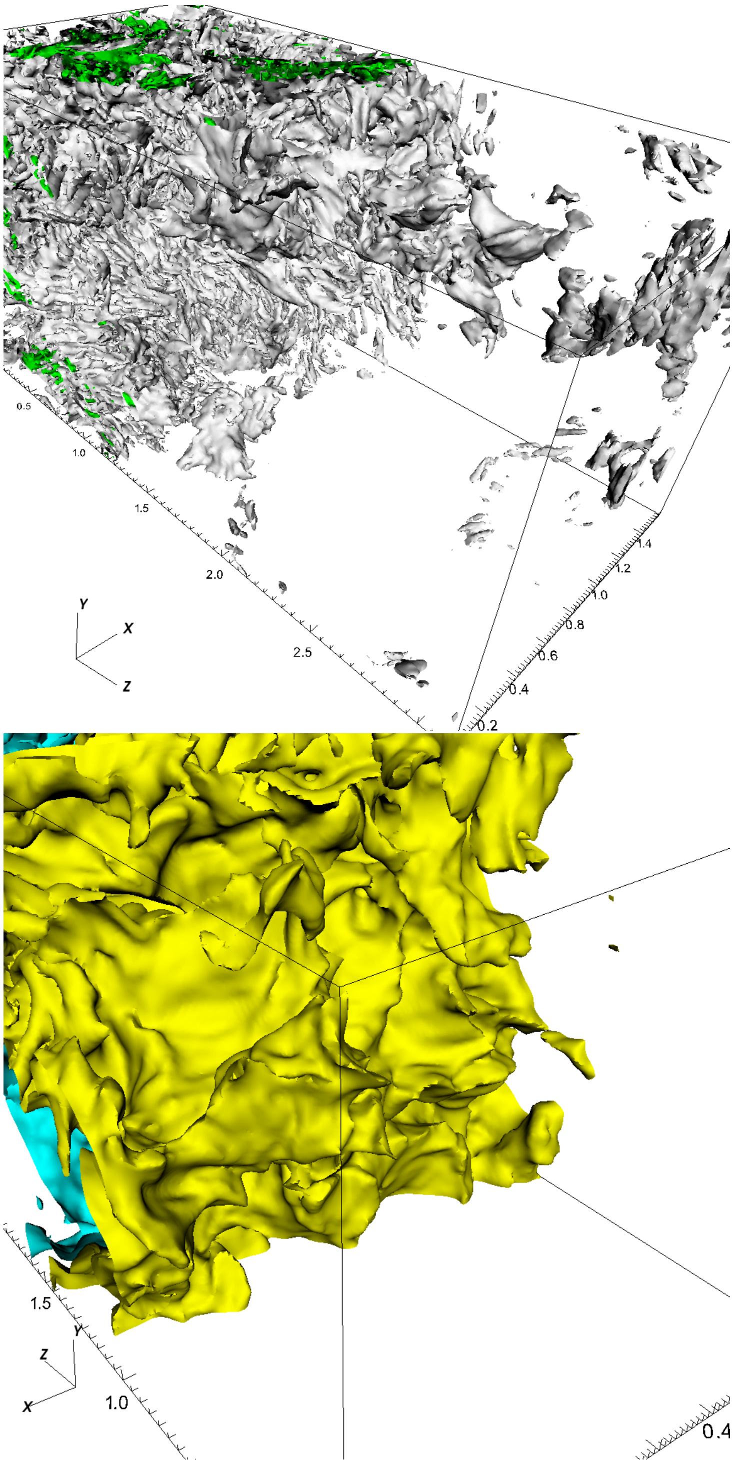


**Figure 1.** Three-dimensional visualization of the kinetic energy and passive scalar fields. Above: isosurfaces of the turbulent kinetic energy, $E = 0.1E_{\max}$ (white) and $E = 0.35E_{\max}$ (green). Below: isosurfaces of the scalar field, $\theta = 0.25$ (yellow) and $\theta = 0.75$ (light blue). The high turbulent energy velocity field is on the right of each image; only a portion of the computational domain has been shown. The images are at $t/\tau = 4$, where $\tau$ is the initial eddy turnover time of the high energy region.

of the velocity field with the same dimensionality, and a stage of evolution with a power law scaling of the mixing thickness is reached in both two and three dimensions. Scalar diffusion in two dimensions is asymptotically faster than in three dimensions, a result already observed in the kinetic energy mixing (see [7, 8]). In three dimensions there is a fair agreement with the wind tunnel experiments on the scalar diffusion from a linear source in a shearless mixing [6, 9].

In the three-dimensional flow, the scalar variance has already reached its maximum after less than one eddy turnover time and, since that instant, it slowly decreases. In the following 10 eddy turnover times, about 20% of the variance present at $t/\tau = 1$ is lost. In two dimensions, the scalar flow is almost twice as large and the initial variance generation last longer: the maximum is attained later and is about 50% higher. The presence of the turbulent energy gradient is instead felt on the distribution of higher order moments as the scalar skewness shown in figures 2.

In contrast with the velocity field, the spatial distribution of the scalar moments shows the presence of two intermittent fronts at the borders of the mixed region. These two fronts are at a distance from the mixing centre approximatively equal to $2\Delta_\theta$.

Because of the gradual thickening of the mixing layer, the kinetic energy and the scalar gradients, which drive the scalar spreading, become milder and milder. As a consequence, not only the scalar flux is reduced, but also the intermittency levels of the two scalar fronts decrease in time. Because of the energy decay, this reduction is faster in three dimensions.

Intermittency is not limited to large scale scalar fluctuations, but is quickly spread to small scale fluctuations, as can be inferred from the skewness and kurtosis of the scalar derivative in the inhomogeneous direction. Two peaks of large intermittency are present in correspondence of the two intermittent fronts since $t/\tau = 1$. However, their time evolution is different from the one seen for large scale scalar fluctuations: the peaks tend to decay in time, but small scale intermittency tends to last longer in the three dimensional case.

In absence of an energy gradient (i.e., $E_1/E_2 = 1$) we observed the same mixing lenght temporal growth but the two scalar intermittent fronts at the borders of the mixing layer become symmetric [7].

In the central part of the mixing layer, between the two intermittent fronts, the scalar fluctuations tend, after few eddy turnover times, to a slow varying state. In two dimensions, the passive scalar exponent in the inertial range is about $-1.4$, which is roughly one half of the $-3$ exponent of the velocity field and is far from the $\kappa^{-1}$ inertial range scaling of homogeneous and statistically stationary flows (see, e.g., [10]). In three dimensions, the difference between scalar and velocity exponents is very mild and both tend to approach the homogeneous turbulence scaling. However, the scalar spectrum seems to show a wider inertial range region, a feature which has been observed also in homogeneous flows at moderate Reynolds numbers, see [11].

## Conclusion

The diffusion length, $\Delta_\theta$, of a passive scalar through a shearless turbulent mixing closely follows the temporal evolution of the self-diffusion length of the velocity field $\Delta_E$. In two dimensions the growth is faster: $\Delta_E \sim \Delta_\theta \sim t^{0.68}$, while in three dimensions $\Delta_E \sim \Delta_\theta \sim t^{0.46}$. The scalar flow is about two times larger in two dimensions than in three dimensions. Also the scalar variance in the centre of the mixed layer is 50% higher in two dimensional case.

An important observation concerns the presence of two scalar intermittent fronts which appear at the sides of the mixed region. The intermittency levels of the fronts is high and gradually decay in time. In all simulations, the fronts move from the initial position of the interface. However, he front toward the high energy side of the mixing shows a deeper penetration and a higher intermittency. This asymmetry is milder in the three-dimensional case. The intermittency is not limited to the large scale, but involves also the small scale. In

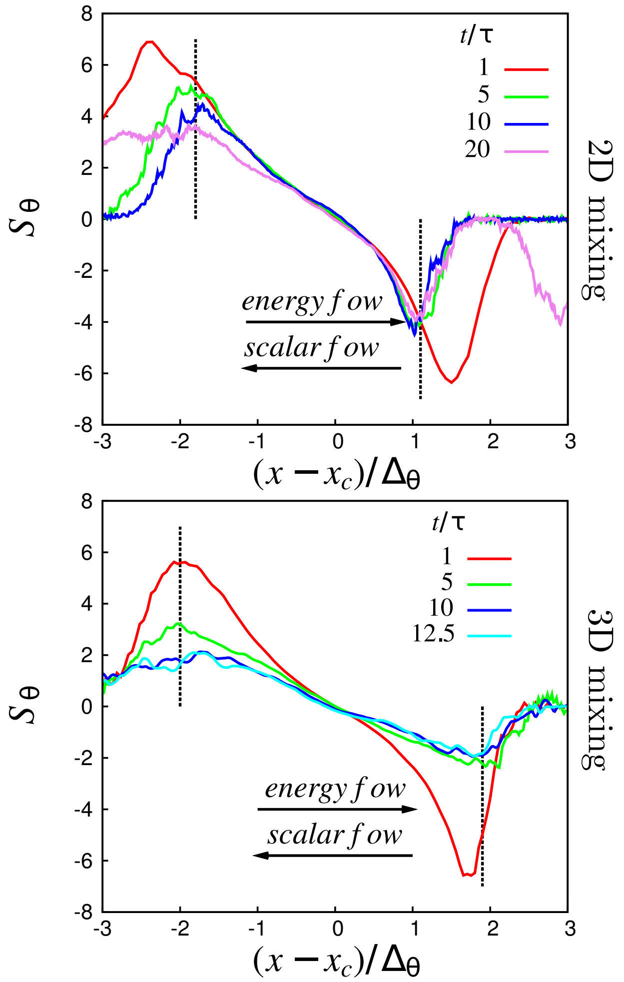


**Figure 2.** Scalar skewness distribution in the simulations with initial energy ratio $E_1/E_2 = 6.7$. The vertical dashed lines indicate the position of maximum intermittence at the end of the simulation.

two dimensions, the small scale intermittency decay is very fast. In three dimensions, it is more intense and lasts longer. The large scale intermittency follows an opposite behaviour.

In the centre of the mixing layer, in three dimensions, the passive scalar and velocity spectra both show an inertial range with an exponent close to -5/3. In two dimensions, the velocity shows a $\kappa^{-3}$ inertial range, while the passive scalar shows a $\kappa^{-1.4}$ inertial range. An exponent value which is closer to the three dimensional $\kappa^{-5/3}$ scaling than to the two dimensional $k^{-1}$ scaling, which is observed in the case of passive scalar transport in homogeneous condition.

## Method

From a numerical point of view, the flow is assumed to be contained in a parallelepiped (or a rectangle in two dimensions) and periodic boundary conditions are applied to all the spatial directions. The initial condition is obtained by the linear match of two homogeneous and isotropic fields with the same integral scale but with a different turbulent kinetic energy. The imposed initial energy ratio is 6.7 in both two and three dimensions. The passive scalar is introduced in the low energy region of the flow at $t = 0$: the scalar concentration is initially uniform in the two isotropic regions, $\theta = 0$ in the high energy region and $\theta = 1$ in the low energy region. The discontinuity is replaced by a sufficiently smooth transition to avoid the Gibbs phenomenon. No scalar fluctuation is introduced in the initial conditions.

In the three-dimensional simulations, the highest turbulent field has a Taylor microscale Reynolds number equal to 250. The directions normal to the energy gradient in this flow configuration remain statistically homogeneous during the decay, so that all the statistics can be computed as plane averages in these directions.

The mass, momentum and and the scalar transport equation are solved by using a dealiased pseudospectral Fourier-Galerkin spatial discretization coupled with a fourth order Runge-Kutta explicit time integration. About 22 initial eddy turnover times have been simulated in two dimensions and 12.5 initial eddy turnover times in three dimensions.

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